SHELL-MODEL LANCZOS-METHOD STUDIES
OF THE GAMOW-TELLER STRENGTH FUNCTION
IN ASTROPHYSICS

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ABSTRACT

We utilize the Livermore system of vectorized shell-model codes to study the Gamow-Teller strength functions for a variety of nuclei whose weak-interaction properties are of interest in astrophysics. The Lanczos method is used to diagonalize a model space consisting of low-seniority excitations (including all states directly populated by the Gamow-Teller operator) with a realistic finite-range effective interaction. The effects of model-space truncation are systematically investigated for each nucleus. The results of these calculations are discussed in connection with problems in stellar evolution, supernova collapse, and nucleosynthesis.

1.   Introduction

Gamow-Teller (GT) transitions play an important role in numerous astrophysical environments. Unfortunately, however, the transitions of interest are often not accessible experimentally. Examples of such transitions are beta decays between thermally populated excited states of nuclei in a hot stellar plasma or the decay properties of nuclei far from stability. Therefore, the best one can do is to estimate the rates of such transitions with a good shell model calculation using a realistic effective interaction in as large a model space as can be accommodated. In this paper we give some examples of the kinds of transitions of interest in different astrophysical environments.

2.   The Method

In order to describe transitions for large numbers of unknown nuclei far from stability we need a prescription to generate the

Gamow-Teller strength functions which is sufficiently general to avoid systematic errors yet not so complex as to be computationally intractable. The following is a brief review of the algorithm which we have developed.

The first task is the construction of a parent state vector, |P>. This we generate by diagonalizing in a space of the minimum number of low-seniority configurations necessary to describe the one-body character of the low-lying states of interest. This typically involves a small basis ($\sim$1-100 Slater determinants (SD's)). Although it would be sometimes useful to include more collective character in these states, the introduction of more complex configurations usually leads to an immense space of multiparticle configurations in the daughter nucleus which is too large to be easily diagonalized. On the other hand, it can often be argued that collective states do not carry much GT strength to the low-lying states of interest in the daughter nucleus. Therefore we neglect the effect of these configurations.

For many applications of interest we must construct a Hamiltonian for unknown nuclei, or for large model spaces for which no phenomenological two-body matrix elements are available. Therefore, we prefer to use a realistic finite-range two-body interaction derived from a G-matrix calculation based upon the free nucleon-nucleon force. For most of the calculations described here we utilize the Kallio-Koltveit[1] interaction which is probably the simplest available approximation to such a realistic force. Although we are experimenting with more sophisticated effective interactions[2,3], at present this interaction seems to be adequate.

Once the parent states have been constructed, we then generate what we call the collective Gamow-Teller state, |CGT>, by operating on the parent state with the GT operator, i.e.,

$$|CGT> = \sigma \cdot \tau^{\pm} |P> \ . \tag{1}$$

This state is not an eigenvector of the system but contains the sum total of all of the GT strength which will be distributed among the eigenstates of the daughter nucleus. The size of the |CGT> is typically about an order of magnitude larger than the parent state, i.e. $\sim$10-1000 SD's. The construction of the |CGT> for the simple case of ^{90}Zr is schematically illustrated in Figure 1.

The |CGT> is useful for two different purposes. For one, we use it as a starting point to generate the basis of states which will be diagonalized to form the spectrum of daughter states. The other purpose is as a projection operator of the GT strength function once the spectrum of daughter states is known.

To generate the basis of states in the daughter nucleus we begin by operating on the |CGT> with the J^2 operator. This operation can have the effect of introducing higher seniority configurations and is necessary to insure that the eigenstates of the basis have good angular-momenta. This operation is repeated until the basis saturates and typically increases the size of the m-scheme basis by about a factor of 20. We also operate on this basis with the isospin exchange operator, τ^2, which can have the effect of

introducing particle-hole excitations into the basis as illustrated in Fig. 1. This operation again increases the size of the basis by about an order of magnitude so that a typical basis size becomes ∿10000 to 100000 SD's. To these congigurations we often find it is necessary to add still a few more low-seniority excitations to account for configurations not produced by the above operations.

Finally, the Lanczos algorithm[4,5] is utilized to diagonalize this space and produce N (N~30) approximate eigenstates. The |CGT> is then used to project out the GT strength from these states, i.e.,

$$S(E) = \sum <CGT|\Phi_i> \exp[-(E-E_i)^2/2\sigma_i^2]/((2\pi)^{1/2}\sigma_i) \quad (2)$$

where, Φ_i, are the approximate Lanczos eigenvectors. The gaussian factor takes into account the calculated dispersion of the eigenvalues for these vectors about their mean energies, E_i.

Figure 2 illustrates a calculation[6] of the strength function for ^{90}Zr for which the entire GT strength distribution is known[7] from (p,n) data. A canonical quenching factor of 0.5 has been assumed and a width has been added to the calculated states to account for the effects of coupling to the background of multi-particle-hole excitations[6]. The overall reproduction of the strength function is very good.
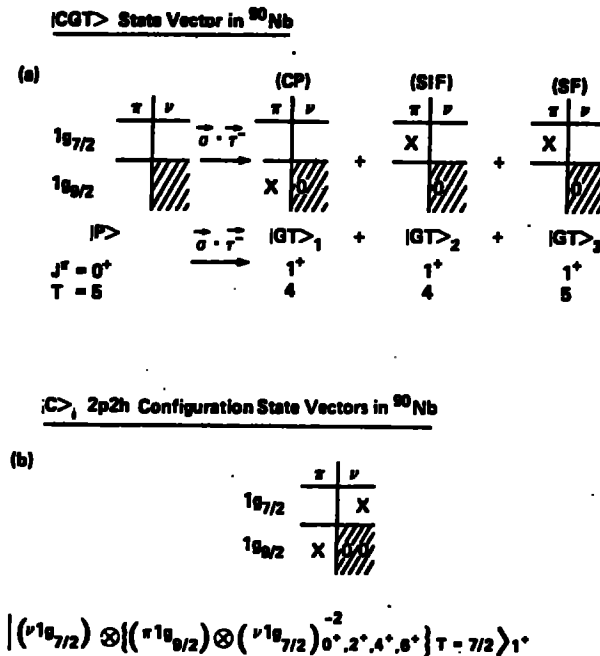

Fig. 1 A schematic illustration of the basis configurations involved in the calculation[6] of the GT strength function for ^{90}Zr. The 2p2h configurations are produced by the operation of τ^2 on the |CGT>.
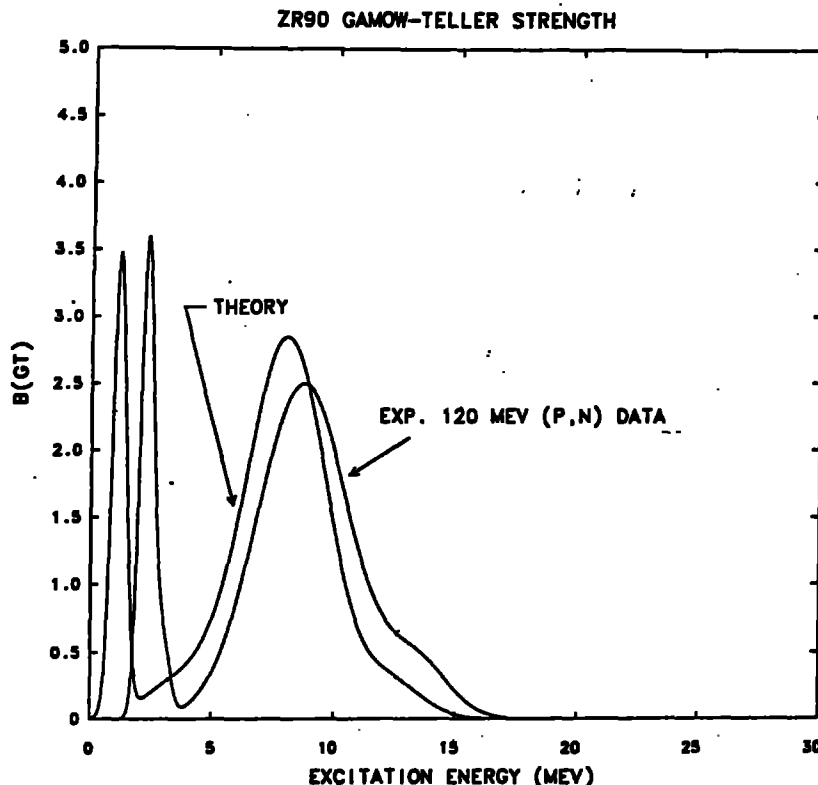
Fig. 2. Comparison of calculated and theoretical GT strength function for $^{90}Zr \rightarrow ^{90}Nb$. A quenching factor of 0.5 is assumed.

3. Gamow-Teller Interactions far from Stability

Beta decay rates of unknown nuclei far from stability play a particularly important role[8-10] in the determination of atomic abundances. About half of the elements heavier than iron are thought to be produced by rapid neutron capture (r-process) in an explosive stellar environment (possibly supernovae though no one knows for sure) in which neutrons are captured on time scales of the order of ms. In the classical formulation of this process, it is particularly easy to see how the beta decay rates determine the final abundances. In this formulation, neutron captures are assumed to be so rapid that $(n,\gamma) \leftrightarrow (\gamma,n)$ equilibrium is established. This leads to the so-called "waiting-point" approximation[8] in which the equilibrium resides on one or two particular isotopes for a given element until beta decay can occur at which time (n,γ) equilibrium is again immediately established at a new waiting point and so on. The abundance of a given element along this path of waiting points will then be given by the solution to the set of coupled differential equations;

$$\frac{dN(Z)}{dt} = \lambda_\beta(Z-1)N(Z-1) - \lambda_\beta(Z)N(Z) \quad , \tag{3}$$

where, λ_β, is the beta-decay rate at the waiting point.
    If this system of differential equations were integrated to

equilibrium, the ratio of abundances would simply be given by the ratio of beta-decay rates, i.e.,

$$\frac{N(Z)}{N(Z-1)} = \frac{\lambda_\beta(Z-1)}{\lambda_\beta(Z)} \tag{4}$$

Thus, it becomes imperative that the beta-decay rates of nuclei far from stability be known as well as possible in order to accurately model this nucleosynthetic process. There are about 1000 such nuclei which must be calculated. In this paper we illustrate one calculation of a strength function far from stability as an example of the kind of difficulty encountered in this task. The nucleus we discuss is ^{95}Rb. Although this nucleus is not actually along the classical r-process path of waiting points, there are good measurements[11] of the Gamow-Teller strength function within the window of energies accessible to beta-decay.

Figure 3 illustrates the magnitude of the problem encountered far from stability, in particular, the large number of orbitals which can participate in a GT transition (in this case 68 of them). All of them must be considered to satisfy the sum rule and to get the right GT strength in the beta-decay window. Figure 4a is an illustration of the total calculated GT strength function. Figure 4b shows an expanded comparison with the beta-decay data. The calculation has been smoothed with the experimental resolution and a quenching factor of 0.5 has been assumed. The calculations seem to adequately represent the data. The task before us now is to calculate the hundreds of nuclei of importance in the r-process.
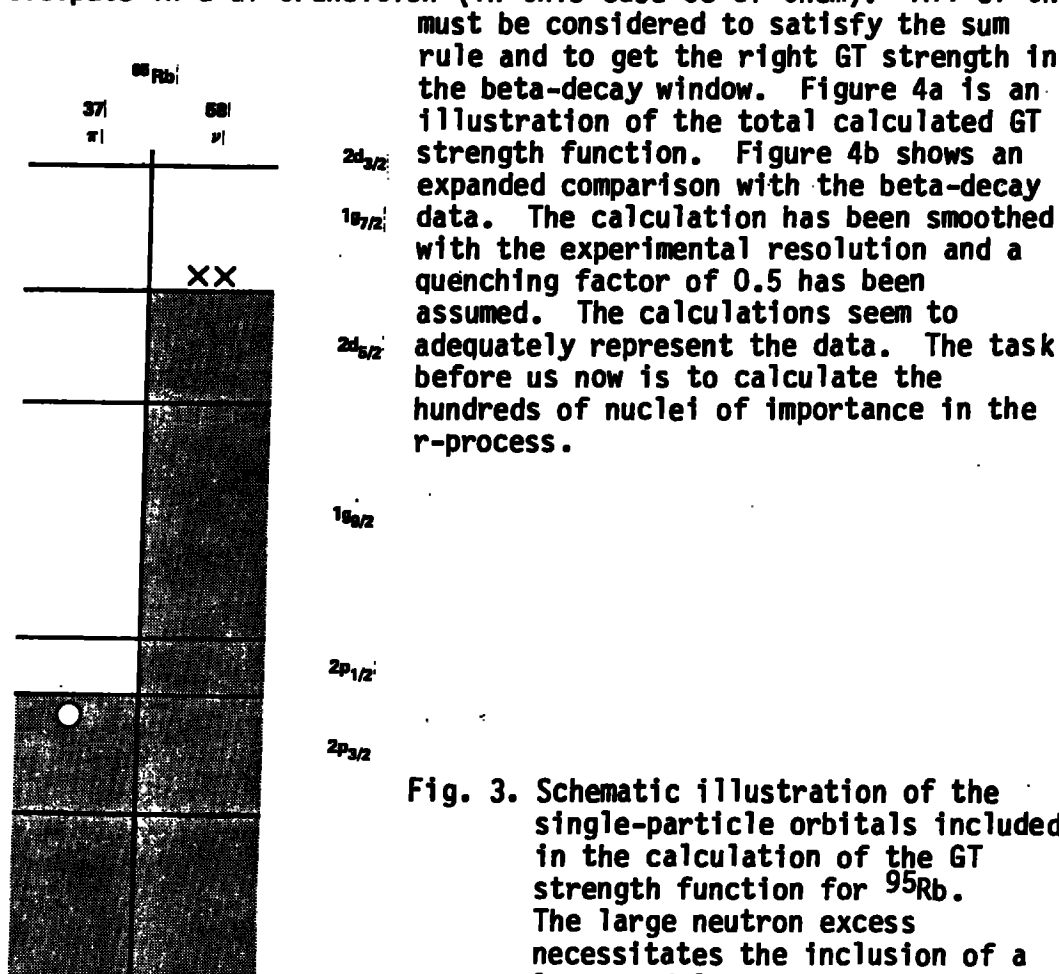

Fig. 3. Schematic illustration of the single-particle orbitals included in the calculation of the GT strength function for ^{95}Rb. The large neutron excess necessitates the inclusion of a large model space.
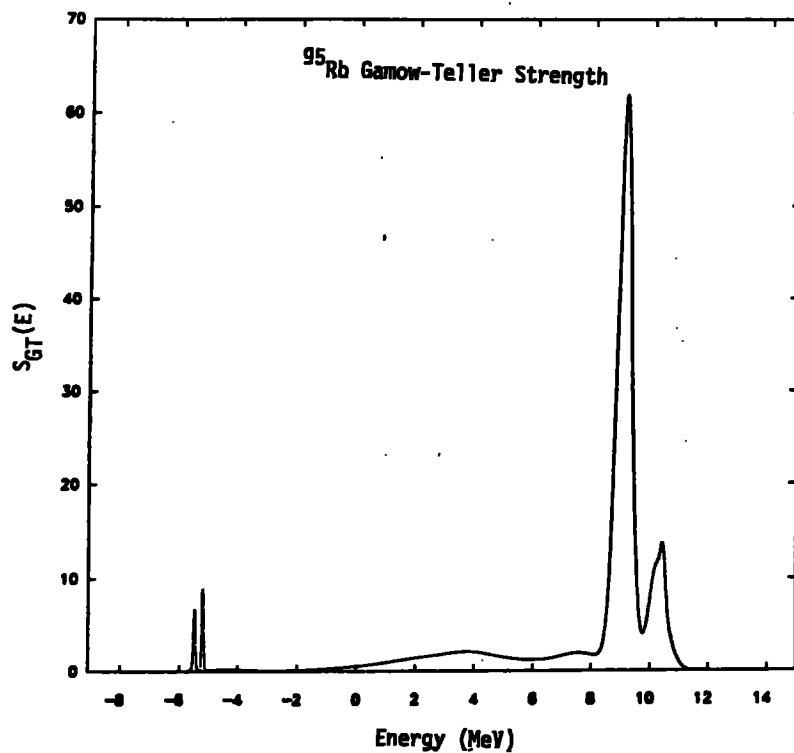
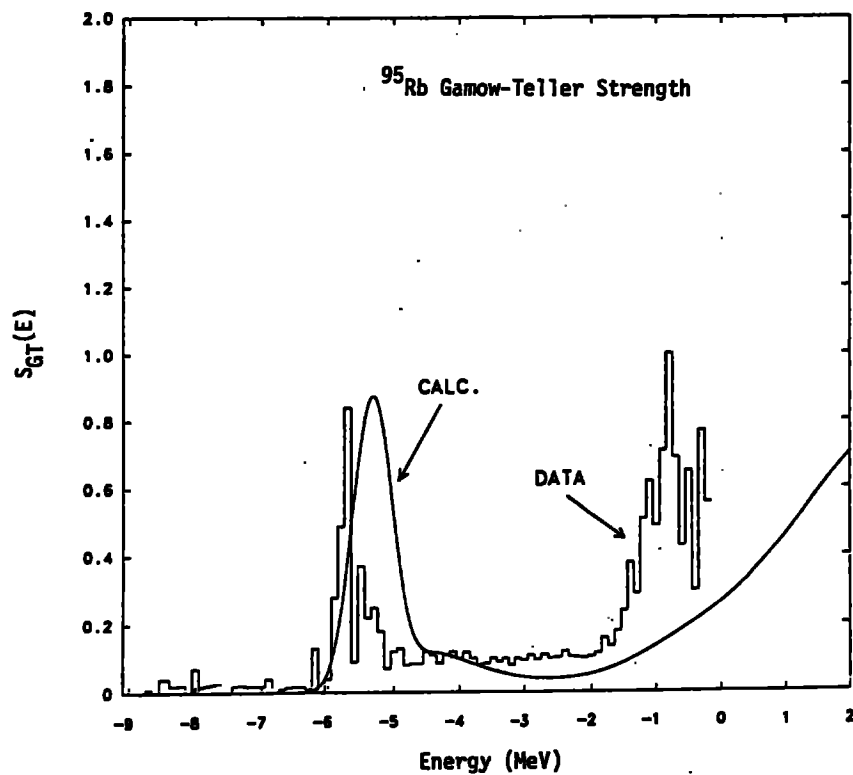Fig. 4a Total calculated GT strength function for ^{95}Rb decay.

Fig. 4b GT strength function for ^{95}Rb decay within the β-decay Q-value window. The data are from ref. 11].

4. Gamow-Teller Interactions Between Excited States

GT interactions involving nuclear excited states occur not only in the r-process but in the quieter environment associated with slow neutron capture (s-process) as well. In the s-process, the neutron-capture time scale is usually long compared to beta-decay time scales so that only nuclei close to stability are involved. A difficulty arises, however in that, at the temperature of the stellar plasma at which this process is thought to occur (kT $\sim$ 30 keV), the thermal population of nuclear excited states can lead to drastically different beta-decay rates which hence affect the nucleosynthesis.

As an example of this effect we show the decay possibilities for ^{99}Tc and its beta-decay half life as a function of stellar temperature in Fig. 5. The terrestrial beta-decay half life of ^{99}Tc is long ($t_{1/2} \sim 2 \times 10^5$y) due to the second-forbidden decay of the $9/2^+$ ground state to the $5/2^+$ ground state of ^{99}Ru. There are, however, excited states at 140 and 181 keV in ^{99}Tc which can have GT allowed transitions to the ground and first excited state of ^{99}Ru. If typical GT-allowed log(ft) values are assumed[12] for these excited states the half life of ^{99}Tc reduces to about 1 yr. at T_9 = 0.35 (kT=30 keV).

This is a dilemma since ^{99}Tc is observed on the surfaces of red-giant stars. In fact this observation[13] was the first definitive proof of ongoing neutron-capture nucleosynthesis in the interior of stars. If this short half life at T_9 = 0.35 is correct, however, the the fact that we see ^{99}Tc at all would seem to indicate that the temperature of the interior is considerably cooler or that the transport time to the surface is extremely fast. For ^{99}Tc to act as such a probe of the stellar environment, however, one must know the GT strength for the unmeasured excited-state transitions as well as possible. If the rates for these transitions are significantly slower than rates based upon typical log(ft) values, a way out of the dilemma exists.

In Fig. 6 we show examples of several calculations in which we have attempted to describe the low-lying states in ^{99}Tc. The energies are difficult to reproduce since these are undoubtedly
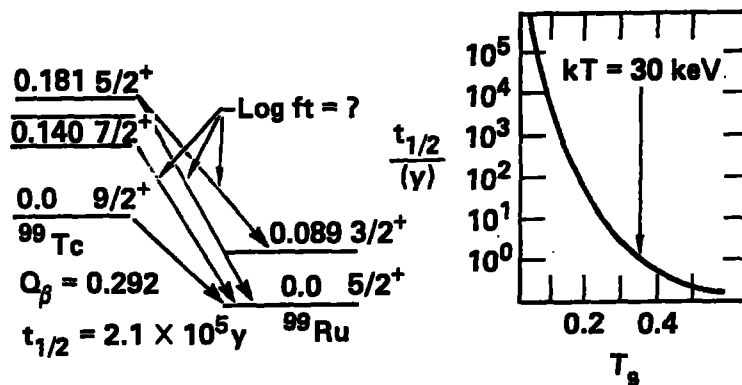

Fig. 5. Levels in ^{99}Tc which can participate in beta decay at stellar temperatures. Also shown are estimated beta-decay half life vs. temperature from ref. 12].

Positive parity states in ^{99}Tc

Fig. 6. Schematic illustration of model spaces used to describe low-lying states in ^{99}Tc.

intruder states pushed down by the effects of coupling the $\pi(1g_{9/2})^3$ configuration to collective excitations of the core. On the other hand, the GT transition strength is probably dominated by the one-body components of these states and is probably adequately described by the limited model space employed here. Nevertheless it would be most useful if the collective component of these transitions could be included (perhaps in the interacting boson model).

5. Gamow-Teller Transitions in a Presupernova Star

Finally, we also mention the importance of good shell-model calculations of the GT strength function for electron-capture transitions in iron-group nuclei. These rates are particularly important for determining the structure of presupernova massive stars. This structure then, in turn, determines whether a core-bounce supernova mechanism can occur. The role of the GT resonance in presupernova electron capture rates and the physics of stellar collapse has been described by a number of authors[14-19].

A presupernova $\sim 25 M_\odot$ star at the end of silicon burning has roughly the structure sketched in figure 7. Nuclear burning occurs in a number of shells in outer envelopes of the star. The inner core contains about a solar-mass of ^{54}Fe and neutron-rich isotopes such as ^{48}Ca, ^{50}Ti, ^{54}Cr and ^{58}Fe. As the core grows in mass it eventually becomes unstable to collapse when it reaches the Chandrasekhar mass ($M \sim 5.8(Y_e)^2$, where Y_e is the ratio of free electrons to baryons). Electron capture rates will have an important effect in determining Y_e and therefore the Chandrasekhar mass.
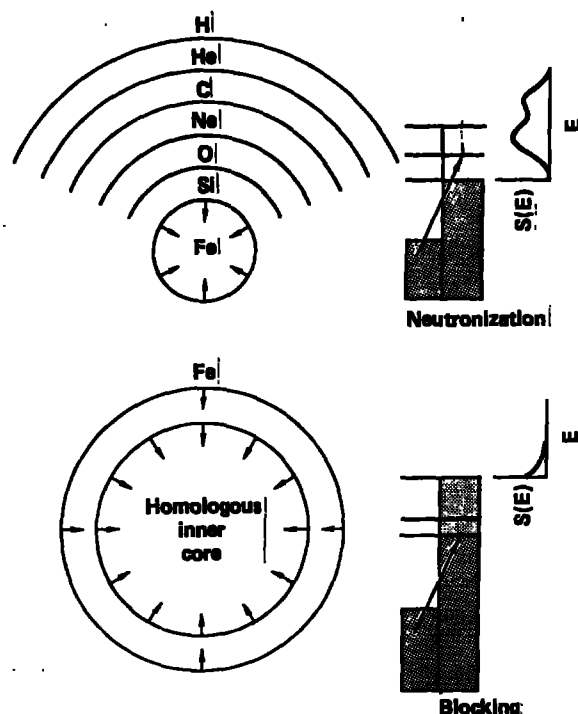
Fig. 7. Schematic illustration of the configuration of a presupernova massive star. GT transitions are rapid during the initial neutronization, but Pauli blocked as the inner homologous core developes.

Figure 8 is an example from some recent calculations[20] of electron-capture GT strength functions for iron-group nuclei. This figure shows a calculation of the strength function for ^{56}Fe calculated in a two-particle two-hole configuration space for both the ground state and the 2^+ first excited state. The GT resonance lies fairly low in energy, ∿2-5MeV, and will participate in the neutronization. This resonance will speed the electron capture rate, and therefore reduce Y_e and the size of the Chandrasekhar mass relative to a calculation which has not included this resonance strength.

This is an important result since it makes the core-bounce mechanism more viable. The reason for this is that the core-bounce is actually experienced by an inner homologous (v∝r) core (see Fig. 7) which then must photodisintegrate the outer core before impinging on outer envelopes of the star. To prevent the complete dissipation of the shock due to photodisintegration, the size of the outer core must be as small as possible, and the size of the inner homologous core must be large. The GT strength function (and the amount of GT quenching) will be important in determining both of these parameters. The total core mass will be small because the presupernova electron-capture rates are fast. On the other hand, the inner homologous core will be large due to the fact that the neutronization
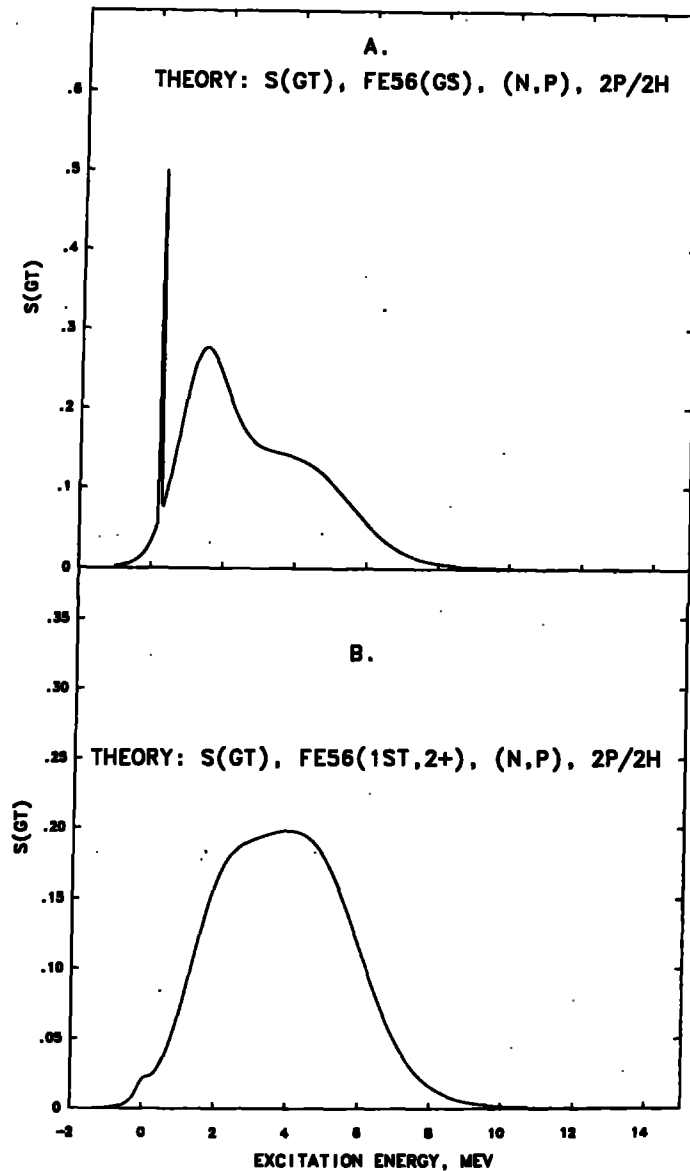
Fig. 8. Calculated[20] GT strength function for electron capture of
the ground and first-excited state of ^{56}Fe.

process will lead to Pauli blocking of the GT transitions as the core
collapses (see Fig. 7). Thus, the inner homologous core does not
benefit from the rapid electron capture rates which minimized the
Chandrasekhar mass.

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